

Computational Geometry

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Exercise 4

We assume that we have a random number generator, $\text{RANDOM}(k)$, which has an integer k as input and generates a random integer between 1 and k in constant time. Now consider the following algorithm:

Algorithm $\text{RANDOMPERMUTATION}(A)$

Input: An array $A[1..n]$.

Output: The array $A[1..n]$ with the same elements, but rearranged into a random permutation.

1. **for** $k \leftarrow n$ **downto** 2 **do**
2. $j \leftarrow \text{RANDOM}(k)$;
3. Exchange $A[k]$ and $A[j]$
4. **od**

Prove that every possible permutation of A is equally likely to be the output of $\text{RANDOMPERMUTATION}(A)$. Also show that the algorithm is no longer correct (it no longer produces every permutation with equal probability) if we change the k in line 2 to n .

Exercise 5

A simple polygon P is called *star-shaped* if it contains a point q such that for any point p in P the line segment $[p, q]$ is contained in P . Give an algorithm whose expected running time is linear to decide whether a simple polygon is star-shaped.